Key

5.2 - Ways to Prove that Quadrilaterals are Parallelograms

A quadrilateral must be a parallelogram if

Thrm 1: both pairs of opposite sides are congruent

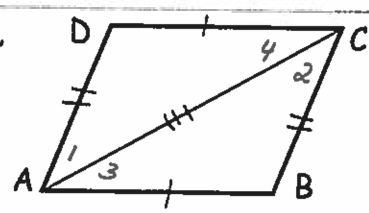
Thrm 2: one pair of sides are both congruent and parallel

Thrm 3: consecutive angles are supplementary

Thrm 4: both pairs of opposite angles are congruent

Thrm 5: the diagonals bisect each other

Thrm 1: If both pairs of opposite sides are congruent, then the quadrilateral is a parallelogram.



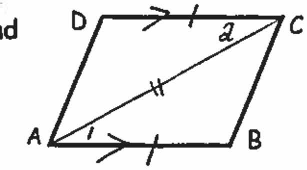
Given: $\overline{AB} \cong \overline{DC}$, $\overline{AD} \cong \overline{BC}$

Prove: ▱ABCD

	Statements	Reasons
1	$\overline{AB} \cong \overline{DC}$, $\overline{AD} \cong \overline{BC}$	Given
2	Draw $\overline{AC}$	Through any 2 points ∃ exactly 1 line.
3	$\overline{AC} \cong \overline{AC}$	Refl. Prop. of =
4	$\triangle ABC \cong \triangle CDA$	SSS ≅ Post.
5	$\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$	CPCTC
6	$\overline{AB} \parallel \overline{DC}$, $\overline{AD} \parallel \overline{BC}$	Alt. Int. ∠s Converse
7	▱ABCD	Def. of ▱

Key

Thrm 2: If one pair of opposite sides are both congruent and parallel, then the quadrilateral is a parallelogram.

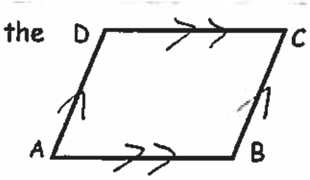


Given: $\overline{AB} \cong \overline{DC}$, $\overline{AB} \parallel \overline{DC}$

Prove: ▱ABCD

Statements	Reasons
1 $\overline{AB} \cong \overline{DC}$, $\overline{AB} \parallel \overline{DC}$	Given
2 Draw $\overline{AC}$	Through any 2 points $\exists$ exactly 1 line.
3 $\angle 1 \cong \angle 2$	Alt. Int. $\angle$s Thrm
4 $\overline{AC} \cong \overline{AC}$	Refl. Prop. of $\cong$
5 $\triangle ABC \cong \triangle CDA$	SAS $\cong$ Post.
6 $\overline{AD} \cong \overline{CB}$	CPCTC
7 ▱ABCD	Both pairs of opp. sides $\cong$ $\rightarrow$ ▱

Thrm 3: If all consecutive angles are supplementary, then the quadrilateral is a parallelogram.



Given: $\angle A$ is supp. to $\angle D$, $\angle A$ is supp. to $\angle B$

Prove: ▱ABCD

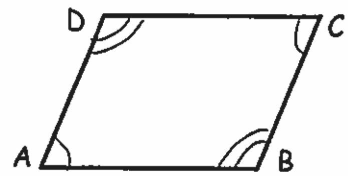
Statements	Reasons
1 $\angle A$ is supp. to both $\angle B$ and $\angle D$	Given
2 $\overline{AD} \parallel \overline{BC}$, $\overline{AB} \parallel \overline{DC}$	S.S. Int. $\angle$s Converse
3 ▱ABCD	Def. of ▱
4	
5	
6	
7	

Key

Thrm 4: If both pairs of opposite angles are congruent, then the quadrilateral is a parallelogram.

Given: $\angle A \cong \angle C$, $\angle B \cong \angle D$

Prove: ▱ABCD

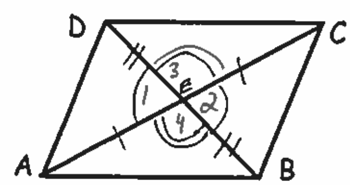


	Statements	Reasons
1	$\angle A \cong \angle C$, $\angle B \cong \angle D$	Given
2	$m\angle A = m\angle C$, $m\angle B = m\angle D$	Def. of $\cong \angle$s
3	$m\angle A + m\angle B + m\angle C + m\angle D = 360°$	Quad. Sum Thrm
4	$m\angle A + m\angle B + m\angle A + m\angle B = 360°$	Subst. Prop. of = (2→3)
5	$2m\angle A + 2m\angle B = 360°$	Distributive Property
6	$m\angle A + m\angle B = 180°$	Division Prop. of =
7	$\angle A$ is supp. to $\angle B$, Likewise $\angle A$ is supp to $\angle D$	Def. of Supp. $\angle$s
8	▱ABCD	All consec. $\angle$s supp. → ▱

Thrm 5: If diagonals bisect each other, then the quadrilateral is a parallelogram.

Given: $\overline{AC}$ and $\overline{BD}$ bisect each other

Prove: ▱ABCD



	Statements	Reasons
1	$\overline{AC}$ and $\overline{BD}$ bisect each other	Given
2	E is the midpt of $\overline{AC}$ and $\overline{BD}$	Def. of seg. bisector
3	$\overline{AE} \cong \overline{CE}$, $\overline{DE} \cong \overline{BE}$	Def. of midpt
4	$\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$	Vert. $\angle$s Thm
5	$\triangle ABE \cong \triangle CDE$, $\triangle ADE \cong \triangle CBE$	SAS $\cong$ Post
6	$\overline{AB} \cong \overline{CD}$, $\overline{AD} \cong \overline{CB}$	CPCTC
7	▱ABCD	Both pairs of opp. sides $\cong$ → ▱

Key

Using Coordinate Geometry:

Using Slope: Lines with the same slope are parallel.
Lines with opposite reciprocal slopes are perpendicular.

$$m=\frac{\Delta y}{\Delta x}=\frac{y_2-y_1}{x_2-x_1}$$

Midpoint of a Segment: The coordinates of the midpoint are the averages of the coordinates of the endpoints of the segment.

$$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

The Distance Formula: $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Consider points A (-2,5) and B (2,7).

Find the slope of $\overline{AB}$.

$m=\frac{\Delta y}{\Delta x}=\frac{7-5}{2-(-2)}=\frac{2}{4}=\frac{1}{2}$

$\boxed{m=\frac{1}{2}}$

Find the midpoint of $\overline{AB}$.

$m\left(\frac{-2+2}{2},\frac{5+7}{2}\right)$

$\boxed{m(0,6)}$

Find AB.

$AB=\sqrt{(2-(-2))^2+(7-5)^2}$

$AB=\sqrt{4^2+2^2}$

$AB=\sqrt{16+4}$

$AB=\sqrt{20}$

$\boxed{AB=2\sqrt{5}}$

Key

Using Coordinate Geometry to Prove a Quadrilateral is a Parallelogram:

Def: both pairs of opposite sides are parallel Use slope

Thrm 1: both pairs of opposite sides are congruent Use the distance formula

Thrm 2: one pair of sides are both congruent and parallel Use the distance formula and slope

Thrm 3: consecutive angles are supplementary X

Thrm 4: both pairs of opposite angles are congruent X

Thrm 5: the diagonals bisect each other Use the midpoint formula

Prove ABCD is a parallelogram using 2 coordinate geometry proofs.

A(-2, 0) B(0, 4) C(6, 6) D(4, 2)

① Show both pairs of opp. sides are //.

$m \text{ of } \overline{AB} = \frac{\Delta y}{\Delta x} = \frac{4-0}{0-(-2)} = \frac{4}{2} = 2$

$m \text{ of } \overline{CD} = \frac{\Delta y}{\Delta x} = \frac{6-2}{6-4} = \frac{4}{2} = 2$

Same slope → $\overline{AB} \parallel \overline{CD}$

$m \text{ of } \overline{BC} = \frac{\Delta y}{\Delta x} = \frac{6-4}{6-0} = \frac{2}{6} = \frac{1}{3}$

$m \text{ of } \overline{AD} = \frac{\Delta y}{\Delta x} = \frac{2-0}{4-(-2)} = \frac{2}{6} = \frac{1}{3}$

Same slope → $\overline{BC} \parallel \overline{AD}$

ABCD is a ▱. [Both pairs of opp sides // → ▱ (Def. of ▱)]

② Show both pairs of opp. sides are ≅

$AB = \sqrt{(0-(-2))^2 + (4-0)^2}$ $BC = \sqrt{(6-0)^2 + (6-4)^2}$ $CD = \sqrt{(6-4)^2 + (6-2)^2}$ $AD = \sqrt{(4-(-2))^2 + (2-0)^2}$

$AB = \sqrt{2^2 + 4^2}$ $BC = \sqrt{6^2 + 2^2}$ $CD = \sqrt{2^2 + 4^2}$ $AD = \sqrt{6^2 + 2^2}$

$AB = \sqrt{4+16}$ $BC = \sqrt{36+4}$ $CD = \sqrt{4+16}$ $AD = \sqrt{36+4}$

$AB = \sqrt{20}$ $BC = \sqrt{40}$ $CD = \sqrt{20}$ $AD = \sqrt{40}$

$AB = 2\sqrt{5}$ $BC = 2\sqrt{10}$ $CD = 2\sqrt{5}$ $AD = 2\sqrt{10}$

Because thier lengths are =, $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{AD}$.

ABCD is a ▱. [Both pairs of opp. sides ≅ → ▱]

Key

Prove JKLM is a parallelogram using 2 coordinate geometry proofs.

J(-6,2) K(-1,3) L(2,-3) M(-3,-4)

① Show the diagonals bisect each other.

midpt of $\overline{JL}$ $\left(\frac{2+(-6)}{2}, \frac{-3+2}{2}\right)$ midpt of $\overline{KM}$ $\left(\frac{-1+(-3)}{2}, \frac{3+(-4)}{2}\right)$

midpt of $\overline{JL}$ $(-2, -\frac{1}{2})$ midpt of $\overline{KM}$ $(-2, -\frac{1}{2})$

The midpoints of $\overline{JL}$ and $\overline{KM}$ are the same so they bisect each other.

JKLM is a ▱ since both diagonals bisect each other.

② Show one pair of opposites sides are ≅ and ∥.

$JK = \sqrt{(-1-(-6))^2+(3-2)^2}$

$JK = \sqrt{5^2+1^2}$

$JK = \sqrt{26}$

$LM = \sqrt{(2-(-3))^2+(-3-(-4))^2}$

$LM = \sqrt{5^2+1^2}$

$LM = \sqrt{26}$

m of $\overline{JK} = \frac{\Delta y}{\Delta x} = \frac{3-2}{-1-(-6)} = \frac{1}{5}$

m of $\overline{LM} = \frac{\Delta y}{\Delta x} = \frac{-3-(-4)}{2-(-3)} = \frac{1}{5}$

$\overline{JK} \cong \overline{LM}$ [Def. of ≅ seg]

$\overline{JK} \parallel \overline{LM}$ [same slope → ∥]

JKLM is a ▱ since a pair of opp. sides are both ≅ and ∥.

Prove RSTU is a parallelogram using 2 coordinate geometry proofs.

R(2,-1) S(1,3) T(6,5) U(7,1)

① Show both pairs of opp. sides are ∥.

m of $\overline{RS} = \frac{\Delta y}{\Delta x} = \frac{3-(-1)}{1-2} = \frac{4}{-1} = -4$ m of $\overline{RU} = \frac{\Delta y}{\Delta x} = \frac{1-(-1)}{7-2} = \frac{2}{5}$

m of $\overline{TU} = \frac{\Delta y}{\Delta x} = \frac{5-1}{6-7} = \frac{4}{-1} = -4$ m of $\overline{ST} = \frac{\Delta y}{\Delta x} = \frac{5-3}{6-1} = \frac{2}{5}$

$\overline{RS} \parallel \overline{TU}$ and $\overline{RU} \parallel \overline{ST}$ since the slopes are equal.

RSTU is a ▱. [Def. of ▱]

② Show both pairs of opposite sides are ≅.

$RS = \sqrt{(2-1)^2+(3-(-1))^2}$ $ST = \sqrt{(6-1)^2+(5-3)^2}$ $TU = \sqrt{(7-6)^2+(5-1)^2}$ $RU = \sqrt{(7-2)^2+(1-(-1))^2}$

$RS = \sqrt{1^2+4^2}$ $ST = \sqrt{5^2+2^2}$ $TU = \sqrt{1^2+4^2}$ $RU = \sqrt{5^2+2^2}$

$RS = \sqrt{17}$ $ST = \sqrt{29}$ $TU = \sqrt{17}$ $RU = \sqrt{29}$

$\overline{RS} \cong \overline{TU}$ and $\overline{ST} \cong \overline{RU}$ [Def. of ≅ seg.]

RSTU is a ▱. [Both pairs of opp. sides ≅ → ▱]